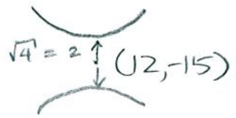
Find the co-ordinates of the vertices and foci, and the equations of the asymptotes of the hyperbola $\frac{(y+15)^2}{4} - \frac{(x-12)^2}{64} = 1$. **State clearly which co-ordinates are for which points.**

SCORE: _____ / 5 PTS



$c^2 = 4 + 64$

$c^2 = 68$

$c = 2\sqrt{17}$

CENTER $= (12, -15)$

VERTICES $= (12, -15 \pm 2) = (12, -13), (12, -17)$ 1 1

FOCI $= (12, -15 \pm 2\sqrt{17})$ 1

ASYMPTOTES: SLOPE $= \pm \frac{\sqrt{4}}{\sqrt{64}} = \pm \frac{2}{8} = \pm \frac{1}{4}$

1 $y + 15 = \pm \frac{1}{4}(x - 12)$

1

Find the equations of the following conics.

SCORE: _____ / 9 PTS

[a] hyperbola with vertices $(-15, 8)$ and $(3, 8)$, and foci $(4, 8)$ and $(-16, 8)$

CENTER $= \left(\frac{-15+3}{2}, 8\right) = (-6, 8)$

$\frac{1}{2}$ $\frac{1}{2}$

$$\frac{(x+6)^2}{81} - \frac{(y-8)^2}{19} = 1$$

1 $1\frac{1}{2}$ $\frac{1}{2}$

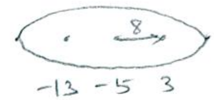
$10^2 = 9^2 + b^2$

$b^2 = 100 - 81 = 19$

+1 FOR CORRECT ORDER OF SUBTRACTION $(x^2 - y^2)$

[b] ellipse with foci $(-13, 10)$ and $(3, 10)$, and major axis of length 18

CENTER $= \left(\frac{-13+3}{2}, 10\right) = (-5, 10)$



$\frac{1}{2}$ $\frac{1}{2}$

$$\frac{(x+5)^2}{81} + \frac{(y-10)^2}{17} = 1$$

1 $1\frac{1}{2}$ $\frac{1}{2}$

$2a = 18 \rightarrow a = 9$

$8^2 = 9^2 - b^2$

$b^2 = 81 - 64 = 17$

Using the distance-based definition of a hyperbola, find the equation of the hyperbola with foci $(0, \pm 9)$ **SCORE: _____ / 9 PTS**
such that the distances from any point on the hyperbola to the foci differ by 6. **Show the algebraic work, not just the final answer.**

IF (x,y) IS ON THE HYPERBOLA,

DISTANCE FROM (x,y) TO $(0,-9)$ − DISTANCE FROM (x,y) TO $(0,9)$ $= \pm 6$

$$\sqrt{x^2+(y+9)^2} - \sqrt{x^2+(y-9)^2} = \pm 6 \quad 2$$

$$\sqrt{x^2+(y+9)^2} = \pm 6 + \sqrt{x^2+(y-9)^2} \quad 1$$

$$\cancel{x^2} + \cancel{y^2} + 18y + \cancel{81} = 36 \pm 12\sqrt{x^2+y^2-18y+81} + \cancel{x^2} + \cancel{y^2} - 18y + \cancel{81} \quad 2$$

$$36y - 36 = \pm 12\sqrt{x^2+y^2-18y+81}$$

$$3y - 3 = \pm\sqrt{x^2+y^2-18y+81} \quad 1$$

$$9y^2 - \cancel{18y} + 9 = x^2 + y^2 - \cancel{18y} + 81 \quad 1$$

$$8y^2 - x^2 = 72 \quad 1$$

$$\frac{y^2}{9} - \frac{x^2}{72} = 1 \quad 1$$

Find the co-ordinates of the foci, vertices, and endpoints of the minor axis of the ellipse **SCORE: _____ / 7 PTS**
$4x^2 + y^2 + 32x + 4y + 32 = 0$. **State clearly which co-ordinates are for which points.**

$$4x^2 + 32x + y^2 + 4y = -32$$

$$1 \quad 4(x^2+8x+16) + (y^2+4y+4) \tfrac{1}{2} = -32 + 4\cdot 16 + 4$$

$$4(x+4)^2 + (y+2)^2 = 36 \quad 1$$

$$\frac{(x+4)^2}{9} + \frac{(y+2)^2}{36} = 1 \quad \tfrac{1}{2}$$

$c^2 = 36 - 9 = 27$

$c = 3\sqrt{3}$

CENTER $= (-4,-2)$

VERTICES $= (-4, -2 \pm 6) = (-4,4), (-4,-8)$ $1\tfrac{1}{2}$

FOCI $= (-4, -2 \pm 3\sqrt{3})$ $1\tfrac{1}{2}$

ENDPOINTS OF MINOR AXIS $= (-4 \pm 3, -2) = (-1,-2), (-7,-2)$

1